Calc 1 Midterm 3 Practice Exam

Question 1. Find the derivative dy/dx for the following:

a) $y = \frac{2x^2}{3\sin(f(x))}$

b) $x\ln(y) = 3x + e^{2y}$

c) $y = (\ln(2x))^{\sin(2x)}$

d) $x^y = y^x$

e) $y = 2x^4 e^{x^2+3x}(3x-5)^{2/3}$

Question 2. Verify that the point $(1/3, 0)$ lies on the curve $e^y = 2y + 3x$, then find the equation of the tangent line to the curve at the point.

Question 3. Two people leave a restaurant at the same time. One drives due north at 50 mi/hr and the other drives due east at 30 mi/hr. After 3 hours, at what rate is their distance from each other increasing?

Question 4. A particle is moving along a hyperbola $xy = 27$. As it reaches the point $(9, 3)$, the y coordinate is decreasing at a rate of -2 cm/s. At what rate is the x coordinate changing at the point $(9, 3)$?

Question 5. Dwayne the Rock Johnson is dropped into a lake, creating a circular ripple. If the circumference of the ripple is increasing at a rate of 3π m/s, at what rate is the area within the circular ripple increasing after:

a) 2s

b) 3s

c) 5s

Question 6. Sketch a continuous graph that satisfies each of the following sets of properties:

a) $f'(x) < 0$ on $(-\infty, 0) \cup (2, \infty)$ and $f'(x) > 0$ on $(0, 2)$

$f''(x) < 0$ on $(1, \infty)$ and $f''(x) > 0$ on $(-\infty, 1)$

$f'(0) = 0$ and $f'(2) = 1$

b) $g'(x) < 0$ on $(-\infty, -1) \cup (3, \infty)$ and $g'(x) > 0$ on $(-1, 3)$

$g''(x) > 0$ on $(-\infty, 3) \cup (3, \infty)$

$g'(1) = 0$ and $g'(3)$ is undefined

Question 7. Find the critical point(s) of $f(x) = x^2e^{2x} + 7$ and identify each as a maximum, a minimum, or neither.

Question 8. Use linear approximation to estimate the following values:

a) $\sqrt{9.001}$

b) $4.1^2 + \frac{1}{4.1}$

c) $\sin\left(\frac{91\pi}{90}\right)$

Question 9) Find the following limits:

a) $\lim_{x\to-\infty} \frac{x^2}{e^{1-x}}$

b) $\lim_{x\to\infty} \left(e^x + x\right)^{1/x}$

c) $\lim_{x\to\infty}\left(x^2\ln(3x)+3\right)$

d) $\lim_{x\to\infty}\left(x\ln\left(1-\frac{7}{x}\right)\right)$

e) $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin(x)}\right)$

f) $\lim_{x\to\infty} x^{1/x}$

g) $\lim_{x\to\pi^+} \sin(x)\ln(x-\pi)$.

Question 10. Show that $\lim_{n\to\infty} A_0\left(1+\frac{r}{n}\right)^{nt} = A_0e^{rt}$.

Question 11 A farmer has 600 m of fencing to section off two adjacent pens on the shore of a river. What is the maximum total area of the two pens?

Question 12. A 2 ft by 2 ft piece of cardboard will be used to make an open-topped box by cutting out squares from each of the corners and folding up the sides. What is the maximum volume possible for the box? What are the dimensions of the box that achieve this maximum volume?

Question 13. Find the point(s) on the circle $x^2 + y^2 = 9$ that are closest to the point $(3, -3)$.

Question 14. A square-bottomed box with no top has a fixed volume of 1000 cm^3. What is the minimum amount of material that can be used to make the box?

Question 15. Find the oblique (slant) asymptote for each of the following:

a) $f(x) = \frac{x^4+3x^2+2x+14}{x^3-2x^2}$

b) $g(x) = \frac{x^2+2}{3x-1}$

c) $h(x) = \frac{x^2+2-e^{-x^2}}{3x-1}$